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Low-Order Approaches to High-Order Systems: Problems and Promises

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Low-order equivalent system approaches to analysis of aircraft pitch dynamics are reviewed. Equivalent systems show promise for reducing the transfer functions of complex, highly augmented aircraft into familiar and interpretable parameters. The merits of such approaches are discussed in light of the flying qualities military specification and the desire to retain the classical criteria for defining acceptable pitch characteristics. Several potential problems with interpreting the equivalent systems parameters are reviewed. Some high-order transfer functions are shown to produce low-order system parameters which may not be equivalent to their classical counterparts and which may be difficult to physically interpret.

Nomenclature

CAP = control anticipation parameter
F_s = longitudinal stick force
g = acceleration due to gravity
h = altitude
HOS = high-order system
K = transfer function gain
LOES = low-order equivalent system
M = aerodynamic pitching moment divided by pitch moment of inertia
M_i = $\partial M/\partial i$ where $i=\delta_e, q, w$, or α
n_z = normal acceleration
q = pitch rate
s = Laplace operator, $\sigma \pm j\omega$
T_2 = first-order lag time constant
T_{θ_2} = time constant of lag between flight path and attitude responses
U_0 = steady-state velocity
w = vertical velocity
Z = aerodynamic force along vertical flight path axis
Z_i = $\partial Z/\partial i$ where $i=\delta_e$ or w
α = angle of attack
γ = vertical flight path angle
δ_e = elevator deflection
ζ_e = damping ratio of equivalent short-period mode
ζ_{sp} = damping ratio of basic vehicle short-period mode
θ = pitch attitude
τ = time delay
τ_e = time delay of equivalent system
ω_e = frequency of equivalent short-period mode
ω_{sp} = frequency of basic vehicle short-period mode
ω_3 = frequency of second-order lag
(a) = $(s+a)$
$[\zeta, \omega]$ = $[s^2+2\zeta\omega s+\omega^2]$

Introduction

THE use of complex augmentation systems for airplanes has necessitated development of some method for predicting and analyzing their handling characteristics. The familiar Military Specification for Flying Qualities of Piloted Airplanes (MIL-F-8785B)[1] was developed in the 1960s for unaugmented airplanes and did not account for high-order augmentation systems (HOS). As a result, some modern airplanes were designed without direct benefit of the guidelines of 8785B since these guidelines were not considered to be applicable. The current specification, MIL-F-8785C,[2] does acknowledge the existence of high-order systems.

A modification of 8785C into a Military Standard and accompanying Handbook[3] is currently underway. It is recognized that a significant contribution of this modification will be a recommendation of methods for specifying flying qualities requirements for HOS. One of the most well-known and most promising methods is the lower-order equivalent systems (LOES) approach.[4-6] This approach would make use of the substantial data base[7] that exists for unaugmented ("classical") aircraft and would retain the requirements of MIL-F-8785C.

This paper will address the promising advances made in specifying longitudinal short-period flying qualities for HOS, and will discuss some unresolved questions and shortcomings which have been encountered with the LOES approach. First, however, the short-period requirements of MIL-F-8785C will be reviewed briefly from the viewpoint of equivalent systems.

Equivalent Systems and MIL-F-8785C

Reference 2 was the first of the military flying quality specifications to acknowledge the existence of equivalent systems approaches to high-order systems: "The contractor shall define equivalent classical systems which have responses most closely matching those of the actual aircraft. Then those numerical requirements . . . which are stated in terms of linear system parameters (such as frequency, damping ratio and modal phase angles) apply to the parameters of that equivalent system rather than to any particular modes of the actual higher-order system." This is implicit acknowledgment that the resulting equivalent dynamic parameters are directly relatable to their classical (i.e., unaugmented) counterparts. The following discussion will briefly review the short-period requirements of 8785C and the philosophy behind their development.

Piloted Longitudinal Control

The primary means of pilot control is stabilization of pitch attitude θ with the elevator δ_e. Good attitude control is essential and is the normal inner loop adopted for other tasks[8,9] such as control of flight path and airspeed. Figure 1 illustrates in block diagram form the closure sequence

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adopted by the pilot for flight path control. Flight path corrections are seen to be made by adjusting pitch attitude in proportion to the flight path error. Flight path is directly related to attitude by

$$\frac{\gamma}{\theta} = \frac{1}{(T_{\theta_2}s+1)} \tag{1}$$

so $1/T_{\theta_2}$ (which also is the zero of the short-period approximation to the θ/F_s transfer function, Fig. 1) represents the lag between attitude response and flight path response.

MIL-F-8785C Requirements

The applicable requirements on equivalent damping ratio ζ_{sp} and equivalent time delay τ are straightforward; Fig. 2 illustrates the 8785C limits for these parameters in terms of flying qualities levels[2] and corresponding Cooper-Harper pilot ratings.

Reference 2 specifies limits on ω_{sp} in terms of the so-called control anticipation parameter (CAP).[10] Figure 3 shows the category A requirements. CAP was derived on the hypothesis that a pilot's control of flight path (in this case represented by normal acceleration, which is related to γ by $n_z = -U_0/g\, s\gamma$) is based upon his sensing the initial pitch acceleration ($\ddot{\theta}_0$) response to elevator deflection. Hence, $\text{CAP} = \ddot{\theta}_0/n_{z_{ss}}$, and it can be shown[7] that for conventional aircraft,

$$\text{CAP} \approx \frac{\omega_{sp}^2}{n/\alpha} \tag{2}$$

where

$$\omega_{sp}^2 \approx Z_w M_q - M_\alpha \tag{3}$$

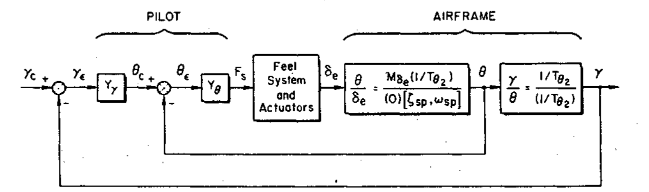
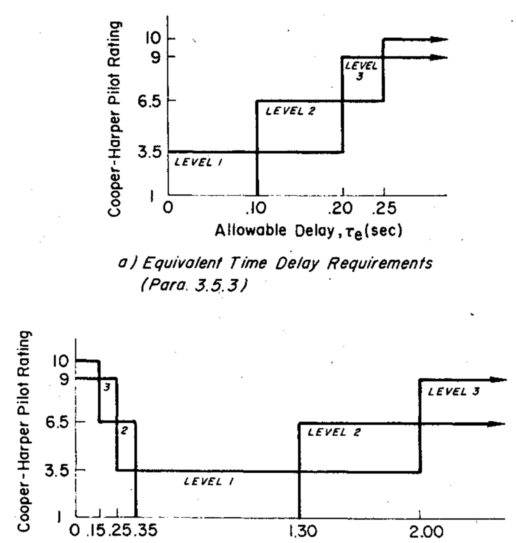


Fig. 1 Block diagram representation of pilot control of pitch attitude and flight path.



a) Equivalent Time Delay Requirements (Para. 3.5.3)

b) Equivalent Damping Ratio Requirements (Para. 3.2.2.1.2, Cat. A and C)

Fig. 2 MIL-F-8785C[2] limits for equivalent time delay and damping ratio in terms of Cooper-Harper pilot ratings.

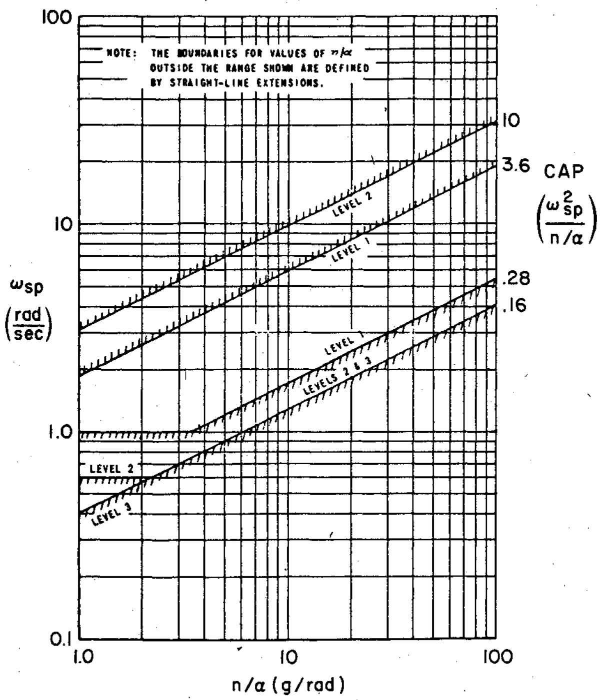


Fig. 3 MIL-F-8785C[2] equivalent short-period frequency requirements (Para. 3.2.2.1, category A).

$$\frac{n}{\alpha} = \frac{U_0}{g}\frac{1}{T_{\theta_2}} \approx \frac{U_0}{g}\left(-Z_w + \frac{Z_{\delta_e}}{M_{\delta_e}}M_w\right) \approx -\frac{U_0}{g}Z_w \tag{4}$$

These simple relations allow us to interpret the lines of constant CAP in Fig. 3 in two separate (but equivalent) ways. Low values of CAP indicate sluggish pitch control and relatively rapid path responses to attitude changes. Hence, the unwanted attitude excursions which occur with sluggish pitch control will result in large path excursions, i.e., path tracks attitude and attitude control is poor. Physically the lower boundary of CAP in Fig. 3 indicates that there is a required consonance between pitch response to the elevator and the path response to pitch changes. The significance of the upper boundary is not as clear and, in fact, the data supporting this boundary are somewhat questionable. We do know, however, that an excessively abrupt pitch response is not acceptable. The upper boundary of CAP corresponds to large overshoots in pitch *rate* which are characterized by the pilot as "abruptness."

With these considerations, the axes of Fig. 3 were chosen for MIL-F-8785C. However, as stated earlier, the approximations applied to derive Eqs. (2-4) assume *conventional* aircraft. The moving-base simulation of A'Harrah and Lockenour[11] (the "supersonic carrier" study) utilized a very *unconventional* design to investigate the independent effects of $1/T_{\theta_2}$ and n/α. The simulation involved a carrier approach and landing in which both airplane speed and carrier speed were varied over a wide range while closure speed remained constant. This allowed for independent variation of n/α and $1/T_{\theta_2}$ but not of the pilot's perception of the task. Results of the simulation are shown in Figs. 4 and 5.‡

‡This simulation[11] included a controlled-incidence wing (i.e., gearing between wing and fuselage varied independently). The published summary data plots included the effects of this variable on pilot rating. The data of Figs. 4 and 5 are previously unpublished results, supplied by the authors, for those cases where the wing and fuselage gearing were equal.

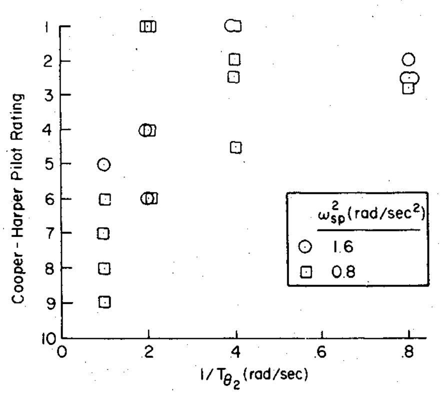


Fig. 4 Variations in pilot rating with $1/T_{\theta_2}$ (data from study of Ref. 11).

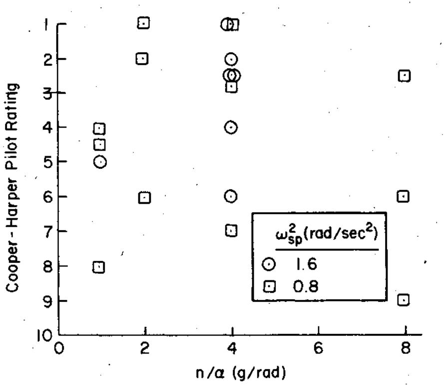


Fig. 5 Variations in pilot rating with n/α (data from study of Ref. 11).

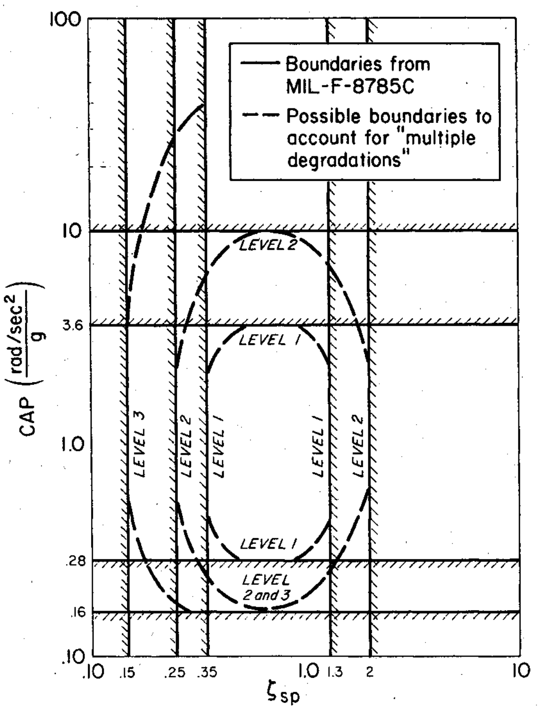


Fig. 6 Alternative specification for equivalent short-period dynamic response.

Figures 4 and 5 show the degradations in Cooper-Harper pilot ratings as $1/T_{\theta_2}$ and n/α, respectively, are decreased. U_0/g is varied by 2.5-80 for the data shown. Figure 4 shows a clear preference for $1/T_{\theta_2}$ greater than 0.2 rad/s, while there is no strong evidence of a preferred value of n/α in Fig. 5. Large scatter in pilot rating is also indicated in Fig. 5. As A'Harrah and Lockenour state, "The fact that the pilot rating is essentially constant with n/α even though the configurations vary from 'satisfactory' to 'unacceptable,' conclusively shows that n/α is not a fundamental acceptance parameter." Again, the vehicle modeled was highly unconventional; but it raises the question of the universality of n/α in Fig. 3. The sensitivity to $1/T_{\theta_2}$ shown in Fig. 4 suggests that $1/T_{\theta_2}$ is a more appropriate criterion. This is as expected since, as discussed above, the lag between flight path and attitude is a major factor in piloted control.

Since the 8785C frequency requirements of Fig. 3 are based upon lines of constant CAP, the specification could be simplified and improved by cross-plotting CAP and ζ_{sp}, as in Fig. 6 (this may necessitate some separate upper and lower limits on ω_{sp} to account for the "shelves" of Fig. 3 at 0.6 and 1.0 rad/s).

Such a cross-plot of two of the three short-period parameters (τ being the third) opens the possibility of accounting for "corners" which exist in MIL-F-8785C—e.g., the instances where a combination of marginal (but Level 1) ω_{sp} and marginal (but Level 1) ζ_{sp} might produce unsatisfactory flying qualities. Such "multiple degradations" are not presently covered, but at least some of the "corners" could be rounded as suggested by dashed lines in Fig. 6.

CAP and Equivalent Systems

Most of the applications of equivalent systems in recent literature[4,5,12] have involved matching in the frequency[4,5] or time[12] domain of the θ/F_s (or $\dot{\theta}/F_s = s\theta/F_s$) transfer function representation of the HOS. The LOES form is

$$\frac{\theta}{F_s} = \frac{K_\theta (1/T_{\theta_e}) e^{-\tau_e s}}{(0)\,[\zeta_e, \omega_e]} \tag{5}$$

where the e subscript reflects the fact that these are equivalent systems parameters. In Neal and Smith's report,[12] $1/T_{\theta_e}$, ζ_e, ω_e, and τ_e were all varied to obtain a good LOES match; while Hodgkinson et al.[4,5] held $1/T_{\theta_e}$ fixed at the aircraft value (i.e., $1/T_{\theta_e} \doteq -Z_w$) unless the mismatch between HOS and LOES necessitated varying it to improve the match.

As we have shown, attitude closure is an all-important first step in piloted control; however, a flight path loop is also normally closed by the pilot (Fig. 1). It should, therefore, be apparent that to obtain an LOES which represents those parameters which 1) the pilot is using as cues, and 2) are the bases for the MIL-F-8785C requirements, a simultaneous fit of the attitude [Eq. (5)] and flight path [Eq. (6)] transfer functions must be performed,

$$\frac{\gamma}{F_s} = \frac{K_\gamma e^{-\tau_e s}}{[\zeta_e, \omega_e]} \tag{6}$$

[Alternative and equally correct forms of Eq. (6) could be $n_z/F_s = (-U_0 s)\gamma/F_s$ or $h/F_s = (U_0/s)\,\gamma/F_s$.] The result, of course, will be very close to that obtained with $1/T_{\theta_e}$ set to the aircraft value of $1/T_{\theta_2}$. In fact, a very good LOES representation of the HOS can be obtained by matching only θ/F_s but restraining $1/T_{\theta_e}$ to the aircraft value of $1/T_{\theta_2}$.

An illustration of the application of these match techniques is given in Table 1. A match of θ/F_s alone with $1/T_{\theta_e}$ free produces $1/T_{\theta_e}$ far from the actual value, but simultaneous matching of θ/F_s and n_z/F_s results in $1/T_{\theta_e}$ almost identical to the airplane value. In fact, the dynamics of the final LOES are very similar for all matched transfer functions except the $1/T_{\theta_e}$-freed fit of θ/F_s alone.

Promises of Equivalent Systems

The available data base of HOS flying qualities piloted evaluations is currently extremely limited [the Neal-Smith[12] and landing approach high-order system (LAHOS)[13] studies

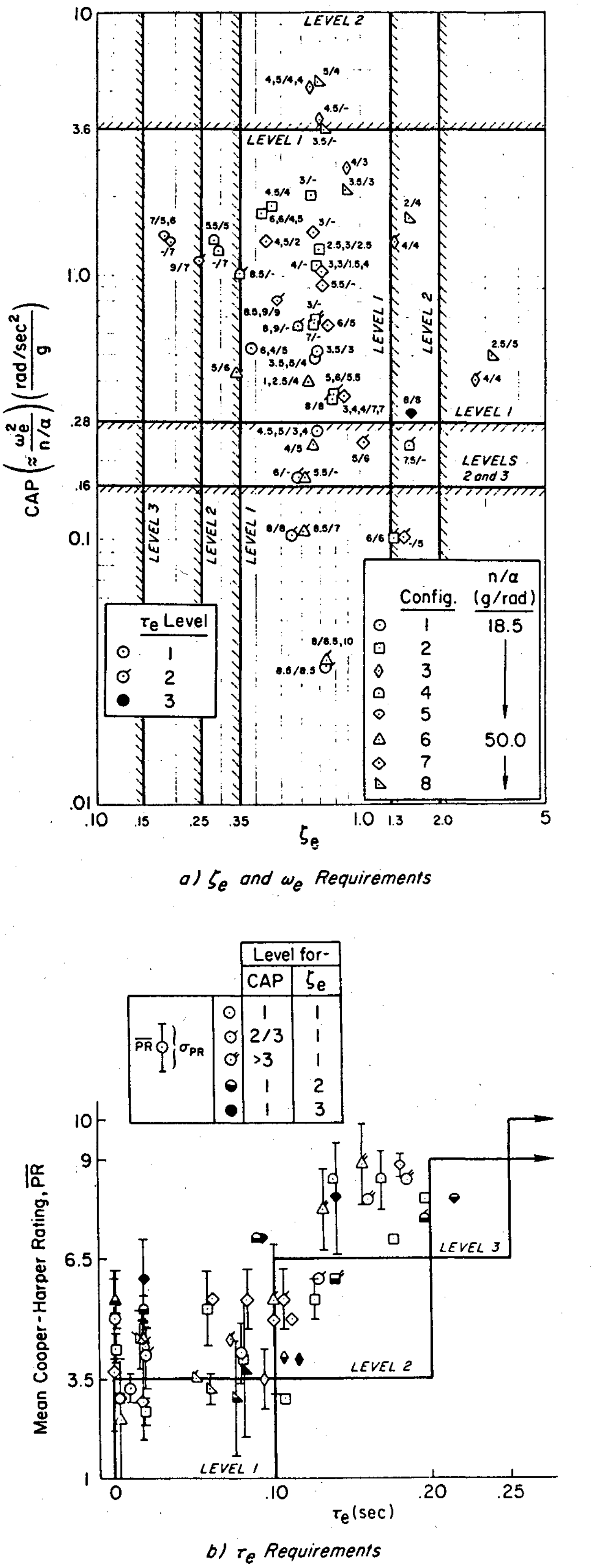


Fig. 7 Comparison of $\dot{\theta}/F_s$ LOES ($1/T_{\theta_2}$ fixed) dynamics with MIL-F-8785C; category A, Neal-Smith[12] configurations and MCAIR[14] matches.

are the most widely recognized]. Fortunately, this small quantity of data covers quite a range of flight conditions and tasks. In this section the LOES parameters ζ_e, ω_e, and τ_e for these data will be compared with Cooper-Harper pilot ratings and with the short-period requirements of MIL-F-8785C.

Neal-Smith Data

The in-flight simulation conducted by Neal and Smith[12] allows a comparison of LOES dynamics with the category A requirements of 8785C. Figure 7 shows the equivalent dynamics[14] of the Neal-Smith configurations and corresponding Cooper-Harper ratings for two pilots. The frequency and damping ratio limits have been cross-plotted as discussed earlier—e.g., Fig. 6—to facilitate presentation of the data. Figure 7a includes actual pilot ratings for each pilot; for those cases which have $\tau_e \leq 0.10$ (Level 1), correlation is quite good. There is clearly a relationship between τ_e and pilot rating (PR) (Fig. 7b), although the τ_e limits appear to be too lenient, since many of the configurations which are predicted to have Level 1 ζ_e and ω_e have higher (poorer) ratings than predicted by τ_e alone. In Fig. 7b, only the mean pilot rating and standard deviation have been plotted to reduce the number of data points.

A point-by-point comparison of the LOES data in Fig. 7 with 8785C shows that the flying qualities levels are accurately predicted for about two-thirds of the configurations. (This requires some liberal judgment—e.g., if a PR change of one-half rating would improve correlation, the configuration is assumed to fit the criterion. Such a PR variation is well within the range of normal ratings variations.) This correlation rate is not outstanding, but is close to that found for flying qualities data in general (for example, the data used to define the ζ_{sp} and ω_{sp} boundaries[7]). It should be noted that correlation could be improved significantly if the ζ_e minimum and maximum Level 1 boundaries were increased and the τ_e boundaries adjusted. The former is examined in a following section on the problems with equivalent systems.

LAHOS Data

Equivalent system matches[5] to the Smith LAHOS[13] data, involving category C (approach and landing) operations, show very good correlation with the 8785C requirements (Fig. 8). In fact, the flying qualities of about 85% of the LAHOS configurations are accurately predicted. The only area of poor correlation in Fig. 8 involves those configurations which should have Level 1 flying qualities, but are rated by the pilots as Level 2. This may be in part a function of the fidelity of the tests and the realism of the tasks: a combination of instrument and visual approaches through touchdown and landing, or with intentional go-around maneuvers. Most of the data upon which the short-period requirements are based[7] were generated for approach and go-around tasks only, seldom including actual landings. The LAHOS data may therefore be more representative of flying qualities in the terminal phases of flight.

One shortcoming of LAHOS is that the equivalent systems do not cover a wide range of ζ_e and ω_e (Fig. 8a); these are Level 2 or worse for only 9 of the 46 configurations. LAHOS is primarily an exercise of the τ_e limits (Fig. 8b). (This is *not* a shortcoming of the LOES approach, but an artifact of the range of HOS evaluated in the LAHOS program.)

Importance of Time Delay

Equivalent systems analysis has been valuable in revealing the major role that τ_e plays in flying qualities. Degradation in pilot ratings can be directly related to pure time delay,[15] and the data of Figs. 7 and 8 show the same influence for equivalent time delay.

Equivalent time delay is a strong function of the frequency of added control system first-order (or second-order) lags, as Fig. 9 indicates. The variation in τ_e between configurations at any one value of T_2 (or ω_3) is small, despite variations in airframe values of ζ_{sp} and ω_{sp} by a factor of four. Even a relatively high-frequency first-order lag at 10 rad/s ($T_2 = 0.1$ s, Fig. 9a) significantly increases τ_e. This can have great implications on such simple control system modifications as stick prefilters.

Table 1 Effect of simultaneous path/attitude LOES matching

HOS:

$$\frac{\theta}{F_s} = \frac{1.0[(s/1.25)+1]}{s\left(\frac{s}{2}+1\right)\left[\frac{s^2}{(4.9)^2}+2\left(\frac{0.7}{4.9}\right)s+1\right]\left[\frac{s^2}{(63)^2}+2\left(\frac{0.75}{63}\right)s+1\right]}$$

$$\frac{n_z}{F_s} = \frac{1.0}{\left(\frac{s}{2}+1\right)\left[\frac{s^2}{(4.9)^2}+2\left(\frac{0.7}{4.9}\right)s+1\right]\left[\frac{s^2}{(63)^2}+2\left(\frac{0.75}{63}\right)s+1\right]}$$

LOES match		$1/T_{\theta_e}$	ζ_e	ω_e	τ_{e_θ}	$\tau_{e_{n_z}}$
θ/F_s	Fixed	1.25	0.80	2.56	0.126	
θ/F_s	Free	4.08	0.52	3.80	0.098	
n_z/F_s			0.80	2.56		0.126
$\frac{\theta}{F_s}+\frac{n_z}{F_s}$	Fixed	1.25	0.80	2.56	0.126	0.126
$\frac{\theta}{F_s}+\frac{n_z}{F_s}$	Free	1.32	0.79	2.59	0.124	0.126

Neal-Smith configuration 2H; Cooper-Harper ratings 5,6, 5.5 (MCAIR matches)

Table 2 Lead/lag configurations with Level 1 LOES and Level 2 pilot ratings

Conf.	HOS						LOES			Pilot ratings	
	$1/T_{\theta_2}$	ζ_{sp}	ω_{sp}	$1/T_1$	$1/T_2$	ω_3	ζ_e	ω_e	τ_e	M	W
1A	1.25	0.69	2.2	0.5	2.0	63.0	0.39	3.14	0	6,4	5
1C	1.25	0.69	2.2	2.0	5.0	16.0	0.67	3.02	0.079	3.5,5	4
2A	1.25	0.70	4.9	2.0	5.0	63.0	0.46	5.96	0	4.5	4
2B	1.25	0.70	4.9	2.0	5.0	16.0	0.42	5.67	0.059	6,6	4,5
7A	2.4	0.73	7.3	3.3	8.0	63.0	0.44	8.23	0	4,5	2

Notes:
1) HOS from Neal-Smith[12]; LOES from MCAIR.[14]
2) Equivalent dynamics are Level 1 on MIL-F-8785C limits.
3) HOS form: $\frac{\dot\theta}{F_s} = \frac{K_\theta(1/T_2)(1/T_{\theta_2})}{(1/T_2)[\zeta_{sp},\omega_{sp}][0.75,\omega_3]}$
4) LOES form: $\frac{\dot\theta}{F_s} = \frac{K_\theta(1/T_{\theta_2})e^{-\tau_e s}}{[\zeta_e,\omega_e]}$

Mismatch

Mismatch functions have been developed[4,14] to define the error between an HOS and its "best-fit" LOES. Application of mismatch as a flying qualities metric has thus far not been totally successful, although it has been noted that poor matches are generally associated with poor pilot ratings.[4,5]

Figure 10 compares the HOS and LOES for a good and poor match. The HOS of Fig. 10a (Neal-Smith configuration 5B) is unacceptable (PR of 7) because of very low damping ratio; the HOS of Fig. 10b (Neal-Smith configuration 5E) also shows low damping, but a first-order lag at 0.5 rad/s causes a more rapid falloff in gain and phase. To fit this response, the LOES produces a large τ_e, a high ζ_e, and accompanying large mismatch. This configuration also received very poor pilot ratings (8/8).

The good correlation rates for the Neal-Smith (Fig. 7) and LAHOS (Fig. 8) data suggest that a mismatch metric is not needed; that is, even for a very poor match (Fig. 10b) the LOES parameters can still accurately predict the flying qualities level for the augmented airplane.

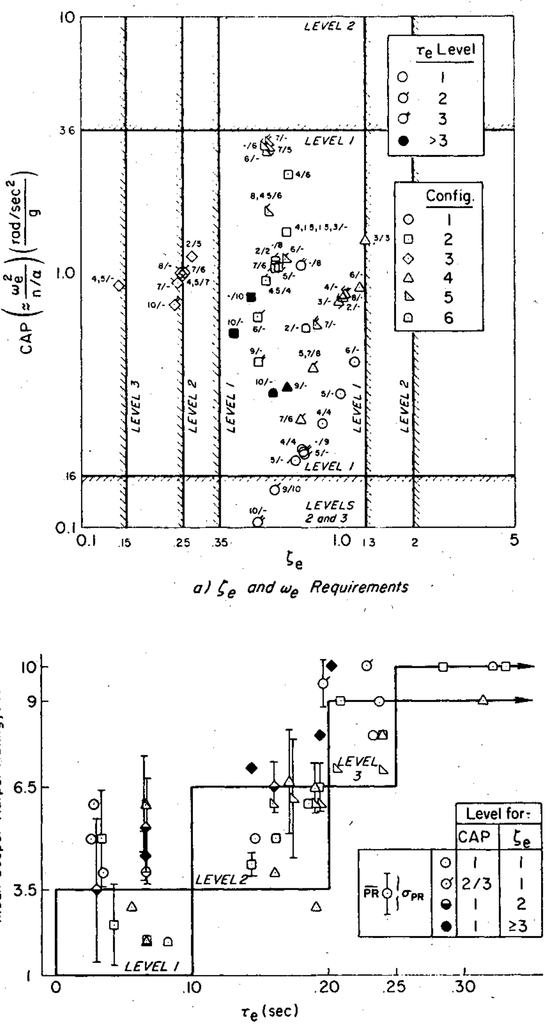
Problems with Equivalent System

The encouragingly positive results from equivalent systems techniques hold much promise for specifying flying qualities for highly sophisticated aircraft. There are, however, a few unresolved questions to be addressed concerning the LOES approach. The most significant question is on the definition of "equivalence" itself. It will be shown that for nonclassical-looking HOS, the LOES parameters are not necessarily equivalent to their classical counterparts.

The Lead Effect

In the previous section, the critical importance of τ_e was discussed. Figure 9 showed that τ_e can be related to existence of an added lag in the frequency range of interest. Of the 51 configurations flown in the Neal-Smith program, 41 were either unaugmented or included only lags (first- or second-order). The flying qualities of 28 of these are correctly predicted by the requirements of MIL-F-8785C. The remaining 10 configurations had combination lead/lag dynamics, and only 5 of these are predicted accurately. For the five which failed, the equivalent dynamics (ζ_e, ω_e, τ_e) predicted Level 1 flying qualities, but they were rated Level 2 by the pilots. Table 2 lists the dynamics of the HOS and LOES for these configurations.

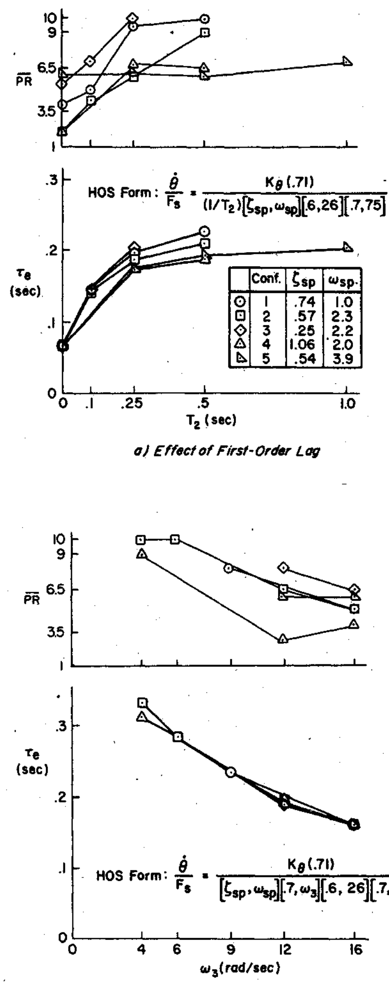
Review of Table 2 reveals some interesting similarities in the low-order equivalent systems. With the exception of configuration 1C, all have $\zeta_e<0.5$ (though still greater than 0.35; these points can be located in the Level 1 region on Fig. 7a). Three have $\tau_e=0$. All but 1C have a first-order lag near



a) ζ_e and ω_e Requirements

b) τ_e Requirements

Fig. 8 Comparison of $\dot{\theta}/F_s$ LOES ($1/T_{\theta_2}$ fixed) dynamics with MIL-F-8785C; category C, LAHOS[13] configurations and MCAIR[5] matches.



a) Effect of First-Order Lag

b) Effect of Second-Order Lag

Fig. 9 Effect of first- and second-order lags on equivalent time delay, LAHOS configurations.

the short-period frequency; 1C has a first-order *lead* near ω_{sp}.

Figures 11 and 12 more clearly show the effects of the added lead/lag combinations on these configurations. In Fig. 11 the Bode amplitude asymptotes of the basic vehicle dynamics ($1/T_{\theta_2}, \omega_{sp}$) of configurations 1, 2, and 7 are sketched and the added dynamics are shown in broken lines. The first-order lead, in all cases, rotates the amplitude ratio counterclockwise by 20 dB/decade at frequencies above the lead frequency, while the first-order lag (near ω_{sp}) serves to rotate the amplitude ratio back. The net effect is an apparent "hump" around ω_{sp}, characterized in the LOES match by a low "equivalent" damping ratio (Table 2).

Similar effects are seen in the phase angle (Fig. 12): the "humps" appear as phase *lead* (since, for the basic configurations, $\tau_e \equiv 0$). In fact, Fig. 12 shows that an LOES match over the frequency range of 0.1-10 rad/s would produce $\tau_e < 0$ (if negative time delays were allowed) for configurations 1A, 2A, and 7A. The small positive τ_e for configurations 1C and 2B results from the relatively low-frequency second-order lag (ω_3) for these cases, 16 rad/s as opposed to 63 rad/s.

There are two potential methods for dealing with lead/lag systems like those of Table 2; unfortunately, neither is physically very appealing. And in each, there is an underlying question as to the universality of the equivalent systems approach.

Redefine Limits on ζ_e

It was mentioned in the previous section that a redefinition of the damping ratio limits of MIL-F-8785C would improve correlations for the Neal-Smith configurations (Fig. 7a). That is, if $\zeta_{e_{min}}$ for level 1 were increased from 0.35 to 0.50, four configurations on Fig. 7a would fit the requirements. Note that these four are tabulated in Table 2 (ignoring configuration 1C, for which none of this discussion is applicable) as our lead/lag violators.

A change in the damping ratio requirements would mean that either: 1) we restrict unaugmented vehicles as well, which is unappealing since $\zeta_{sp_{min}}$ is very well supported by flight test data for classical airplanes[7]; or 2) we specify two sets of requirements—one for unaugmented airplanes and one for augmented airplanes. The latter is especially unattractive, since this is tacit admission that ζ_e is *not* equivalent to ζ_{sp} and that "equivalent systems" is a misnomer. Additionally, it presents the problem of defining an augmented vs unaugmented airplane, e.g., should addition of a simple high-frequency stick filter (whose only major effect is to increase τ_e, Fig. 9) suddenly mean that the airplane must meet a more stringent damping requirement?

Redefine τ_e

As mentioned above, three of the four low-ζ_e violators of Table 2 also have $\tau_e = 0$. As Fig. 12 suggests, a better LOES fit is obtained for these three cases if τ_e is allowed to be less than

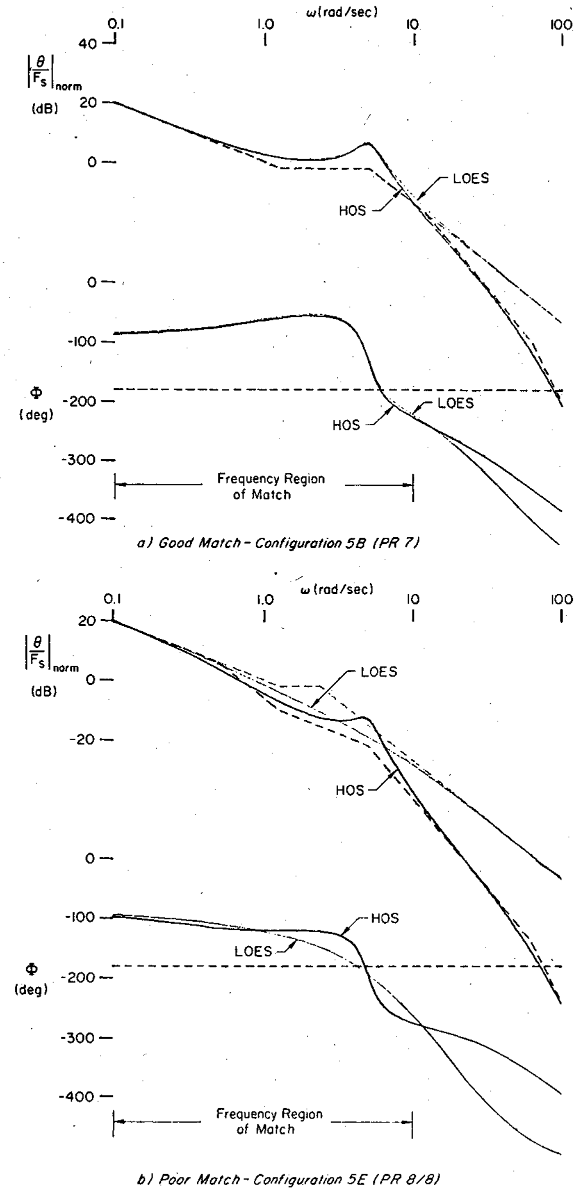


Fig. 10 Examples of good and poor equivalent system matches, HOS from Neal-Smith[12] and LOES from MCAIR.[14]

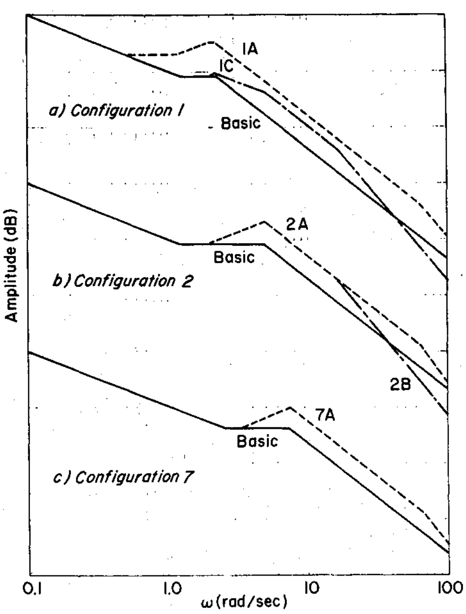


Fig. 11 Comparison of Bode amplitude asymptotes for basic and

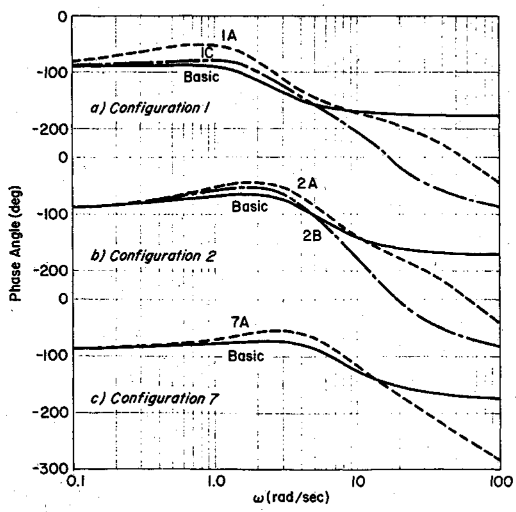


Fig. 12 Comparison of Bode phase angles for basic and augmented configurations of Table 2.

zero. Specifically, negative time delays can be found in an LOES match[16] to be as follows: configuation 1A, $\tau_e = -0.004$ s; configuration 2A, $\tau_e = -0.008$ s; and configuration 7A, $\tau_e = -0.014$ s.

The problem, clearly, is in interpreting the significance of *negative* time delay or time *lead.* Physically, it might be considered to represent a HOS which is too abrupt (i.e., if $\tau < 0$, the system responds to an input τ seconds *before* the input is made or has finite magnitude at zero time).

It is clear that more work must be performed on use of equivalent systems techniques for lead/lag combinations. The encouraging empirical results shown thus far (Figs. 7 and 8) cannot be considered final proof that the equivalent systems approach is universally applicable.

Conclusions

The equivalent systems approach to analysis of highly augmented aircraft continues to show promise. As long as the proper transfer functions are matched, the pitch short-period equivalent dynamics are relatable to their classical counterparts for most cases. The success obtained with recent high-order system flight test results has been encouraging. A key finding has been the significant role that equivalent time delay can play in degradation of longitudinal flying qualities.

Concern has been raised on the applicability of equivalent systems to lead/lag augmentation, especially where the lags occur near the basic vehicle short-period frequency. This area warrants considerable investigation in the future, as it raises the question on whether the "equivalent" dynamics as defined in this paper are really equivalent to their classical counterparts.

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